Mathematics Teacher

THE OFFICIAL JOURNAL OF
THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

VOLUME XXI NOVEMBER, 1928 NUMBER 7

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Published by the
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
LANCASTER, PA. NEW YORK

Entered as second-class matter, March 26, 1927, at the Post Office at Lancaster, Pa., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized November 17, 1927.

THE MATHEMATICS TEACHER

VOLUME XXI NOVEMBER, 1928 NUMBER 7

EDUCATIONAL TESTS—TO STANDARDIZE OR NOT TO STANDARDIZE

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INTRODUCTION

A steamboat should not be condemned because it is not a Rolls-Royce. It was not so intended. In like manner, standardized tests are not to be condemned if they serve adequately the purposes for which they are constructed. A standardized test is an example of one kind of the so-called new-type test, in contrast to the old essay-type of examination, in which some standard performance of so many questions correctly answered in a certain number of minutes has been worked out. No one who has followed the testing movement will question the fact that standardized tests have made certain contributions to education, some of which we shall now enumerate.

CHIEF VALUES OF STANDARDIZED TESTS IN MATHEMATICS

If I may be permitted to do so, I should like to illustrate what I have to say here with reference to the field of mathematics. The chief values of the use of standardized tests in mathematics may be summarized as follows:

1. Such tests have brought out more clearly the problem arising from individual differences in ability among pupils.

2. They have shown us that a great deal of the traditional material is too difficult for most pupils and therefore should not be taught.

3. In some cases they have also shown that certain material was easier than had been expected and that it can be mastered by a large majority of pupils.

4. They have made it possible for the teacher to stop drilling certain pupils beyond the stage of diminishing returns.

5. They have enabled us to develop certain standards of achievement which are clearly defined and which can be assigned to varying levels of intelligence. However, this value has occasionally been overrated.

6. They have contributed to the development of more objective methods of testing.

7. They have, when intelligently used, stimulated pupils to renewed effort in trying to reach certain standards of perfection, or at any rate to improve upon their own previous records.

Three Points of Attack

Although standardized tests have been useful in the past they have many shortcomings as we shall see. My purpose here is not to utterly condemn standardized tests, but rather to point out and emphasize the need for more care in their construction, in their use, and in interpreting the results of their use in the classroom. It is in these three respects that the tests are open to criticism.

Careless Construction of Standardized Tests.—Let us first consider how some of the tests have been constructed. A great deal of the traditional arithmetic and algebra which has been questioned by many teachers and textbook writers as contrary to modern objectives has been replaced by more valuable material. It is an interesting fact, however, that standardized tests have failed to include the latter type of material and thus have made difficult the general acceptance of some of the more modern courses in arithmetic and algebra. In the stage of curriculum building in which we are now engaged it has been repeatedly pointed out that it is far more important for us to get agreement on the fundamentals of a course than it is to standardize mediocrity. As pointed out by Professor Smith,[1] "the complaint is not so much that the tests are solely mechanical, involving only a minimum of intellectual processes—a fault that is probably inevitable in the present stage of development, but which is being successfully removed in some of the arithmetic tests; it is also that the material required for testing the mechanical processes

[1] Smith, David Eugene, "On Improving Algebra Tests," *Teachers College Record*, XXIV, pp. 88–89.

is often such as should play only a minor role, if any, in the education of the average citizen. The tests represent generally a dead level of dull grind, offering to the teacher only this ideal of an algebra course. He may escape from the curriculum, making his own course; he may and should select from the textbook that which he needs for carrying out his plan; but he cannot escape from teaching those things that are required by outside examinations, whether they be set by boards of regents, or by local educational authorities."

A careful study of many of the standardized tests shows:

I. That they have been prepared by people who select material with little regard to the proper objectives to be obtained and who seem to be ignorant of the plans for the reorganization of mathematics. Unless made by those who know how to weigh values, such tests tend to perpetuate the obsolete.

II. That there exist many desirable objectives which were overlooked when they were made. Too many test-makers give no thought to values, and simply measure what is taught without any regard to what is important, that is, some objectives "have no standardized tests by means of which they may be measured." Statistically the tests themselves may be perfect; but when a task is introduced into a test, as has been done in many cases, merely because it represents a certain degree of difficulty on a given scale, it simply tends to prejudice all persons of common sense against the whole movement. For example, if a test-maker has one item that turns out to have a difficulty of 40 and another one of 50, he is too often interested in getting an item that has a difficulty of 45 without any regard to its validity or value in any other sense than that of its difficulty for the pupil.

Examples Illustrating Obsolete Material.—The following example taken from one of the recent arithmetic scales on addition will illustrate what was said above:

The problem has no relation whatever to anything that is real in the life of the child who is called upon to do the task. Moreover, it cannot be justified even from the mental discipline viewpoint in the light of more valuable material that might be offered. The problem is obsolete and fictitious because in the first place no one would ever obtain such decimals as are here represented

113.46
49.6097
19.9
9.87
.0086
18.253
6.04
---

except from measurement; and none except a scientist would need to measure with any degree of precision beyond hundredths. Moreover, even if such accuracy could be obtained and one of the numbers was given correct to ten thousandths, then certainly all of the others should be so represented. Thus, the problem might appear as follows:

113.4600
49.6097
19.9000
9.8700
0.0086
18.2530
6.0400

In no other way could one be sure how much accuracy was claimed for the numbers given. It is the inclusion of such tasks in standardized tests that tends to perpetuate the obsolete in our schools. Teachers have even been known to drill their pupils at length upon such tests to insure their success when the tests are given. If they do not do this, they make up tests like the ones that have been standardized. In such cases improvement in a pupil's score is not necessarily an index of progress in the abilities tested.

Again take one of the examples referred to by Professor Smith in the article referred to above which reads as follows: "How would you find the value one year hence of a war savings stamp for which you now pay $4.16?" In commenting on this Professor Smith says: "The only correct answer possible would be, 'wait one year and see what price I can get for it.' In the examination from which this problem was taken it was probably intended to elicit such a reply, but what the effect on the general type of schools might be it is difficult to say."

What is true of the arithmetic tests is probably true of all the other standardized tests. Take the case of the algebra tests for example. Professor Smith has already pointed out some of the most interesting cases of the perpetuating of obsolete or fictitious examples. I quote from him.[2]

1. Express in simplest form $\sqrt{8}$ and $\sqrt{2/3}$.

This is an interesting inheritance from the sixteenth century, before decimal fractions and tables had come into general use. The first is already in as simple a radical form as it can be, for practical purposes being simpler than $2\sqrt{2}$. The second is in the simplest form for computation when written $\sqrt{0.666}$.

[2] Smith, David Eugene, *loc. cit.*, pp. 90–93.

2. If a cistern can be filled by one pipe in x minutes and emptied by another in $(x+5)$ minutes, what part of the cistern-ful runs in one minute if both pipes are open?

Aside from the question relating to "runs in one minute" (probably a misprint), from the other manifest slip in English, and from the natural doubt as to the value of this ancient though interesting puzzle, the question arises as to whether the tests assist in the improvement of algebra by perpetuating such problems for examination purposes.

3. Simplify $\sqrt{20} \div \sqrt{45} \div \sqrt{1/5}$.

This is an inherited type of example which teachers have been trying to get rid of for a generation past. It is difficult to imagine any real situation that would ever require it.

4. The total number of circus tickets sold was 836. The number of tickets sold to adults was 136 less than twice the number of children's tickets. How many were sold of each?

Is the cause of algebra helped by this style of problem in a test? As an interesting class puzzle it may be commended, but as a type that is to be standardized by these papers, thus being required in schools at large, it is at least not ideal.

5. Under what conditions will $a+b+c=abc$?

If the problem had been "State some condition under which . . . ," it would have been unobjectionable. As it stands, no student who took the examination could possibly have stated "the conditions." It is, therefore, difficult to see how such a test can do anything to assist in putting before schools the real meaning of algebra, or how it tests the ability—of anyone—even of the tester. What are the conditions anyway?

The examples cited above are not unusual. Similar cases can be found in many standardized tests. They would be no worse here than in the textbooks if it were not for the fact that the process of standardization tends to preserve them and to perpetuate their teaching in the schools.

III. Standardized tests are not always adapted to the purpose of educational tests as outlined on pages 378 to 381.

IV. They cannot be easily adapted or modified to meet the needs of different courses of study or of conflicting ideas on what topics should be taught.

V. It is difficult to adapt standardized tests so as to measure objectives which have to be secured in short periods of time.

Misuse of Standardized Tests.—In spite of all the good results mentioned above, no one today would question the statement that the introduction of standardized tests has been accompanied in many places by grave misuse. Although they have been intended to be scientific, they have often been quite the reverse. Sometimes those in charge of the testing program are so unwise as to judge the success or failure of a teacher solely by the outcome of such tests. In other cases of the misuse of test results, invidious comparisons based on the outcome of the tests have not been checked up by a careful study of the methods of teaching employed. Again, certain algebra tests have been regarded as entirely reliable when they were not and the assumption has been made in practice that the tests can be used to measure every feature of the teaching of algebra.

Norms.—Moreover, when definite norms are established there is a temptation for a teacher or a school to be too well satisfied when a class or group of pupils reaches the standard norm of performance. I heard a prominent school official recently congratulating his group upon the fact that they were one above the standard norm in arithmetic. As a matter of fact pupils of the ability of those in his school should have been considerably above the norm. Such attitudes lead to standardization of mediocrity and cause even otherwise good teachers to overlook the fact that the standard norms may be raised by lifting the general level of achievement through better methods of teaching. Someone has said, "The good is enemy of the best." The real value of a pupil's test score as pointed out by Kelley [3] in so far as a given town is concerned lies not in making comparisons with other towns, "state, or national norms, but in knowledge of differences in accomplishment found within the school system" of the particular town in question. He says further that "Ordinarily, extensive grade norms are of no importance in an educational test program, and the lack of published norms, if the test is otherwise suitable, is no hindrance to its complete serviceability in meeting the six major purposes" which he lists for school examination programs.[4] The greatest use of standardized tests in mathematics has been made in arithmetic. This has been due

[3] Kelley, T. L., *Interpretation of Educational Measurements*, World Book Co., 1927, p. 37.

[4] Kelley, T. L., *loc. cit.*, pp. 28–29.

to the fact that the material lends itself readily to standardization. It is only fair to say that even here the tendency today is away from general national standardization and toward practice exercises and diagnosis of individual cases. This has grown out of the realization of the importance of giving to the pupils their standing based upon some definite scale of performance related to their own class rather than trying to place them with reference to a norm based on the performance of some outside group—a practice due entirely to the influence of the recent testing movement.

Dr. Mort pointed out recently to a group of educators that it is no consolation to the father of a dull pupil merely to find out that his child is below median performance. This raises the whole question as to what attitude the teacher of a retarded group should take toward norms of any kind.

The Time Element.—Next comes the matter of the time element. As Dr. Thorndike has often pointed out, the speed with which a child makes errors is of no importance. Then too we need more careful thought with reference to the purpose and place of the so-called "speed" or "time-limit" tests and "power" or "work-limit" tests. Kelley says that[5] "Our knowledge as to the educational and social situations in which speed is of prime importance and those in which power is especially demanded is quite limited. This question is not to be settled by speculation, and relatively few experimental correlation studies comparing the merits of these two functions have been made." The tests we need to use do not presume that every pupil must do a certain amount of work in a given time; they recognize individual differences and needs. If standards are desired in a given class or school, the teacher or teachers in question should set the standard. For all these reasons standardized tests should be used more sparingly and carefully in the future.

Faulty Interpretation of Test Results.—Some of the evils of the faulty interpretation of the results of standardized tests have been implied in what was said above about the misuse of tests. However, the worst evil in the interpretation of such results has been the tendency on the part of teachers and supervisors to judge their pupils adversely on the results shown by giving standardized tests in algebra, for example, which tested

[5] Kelley, T. L., *loc. cit.*, p. 31.

many things which presumably the pupils have never been taught. Not only the pupils themselves but the department of mathematics and even the whole school system have been blamed for what was thought to be a poor showing. As a matter of fact, a properly constituted test based upon what had been taught might have placed such pupils and schools in a favorable light. Such is the reputation that comes to schools who have examinations superimposed upon them with little consideration as to the validity of the tests.

Professor Thorndike's Attitude.—Professor Thorndike in discussing "Standard Tests and Their Use" said:[6] "The testing movement is embarrassed by its success. Persons not qualified by experience or training are making, administering, evaluating and interpreting tests. Caution in conclusions and careful checking against criteria already established are important needs when test results are used to change methods and procedure or to decide the placement of pupils.

"From the research field we should seek better tests with more numerous forms than are now available. Improvement in the instruments of measurement, both of intellect and of school achievement, is more desirable than multiplication of their number. Norms need revision because of changed school conditions and teaching technique. Test items need revaluation or revision. Ambiguous and invalid elements must be discarded, and substitute elements carefully standardized. Unreliability must be ironed out by rigorous experimentation. It requires time, labor, and money, as well as acumen, to accomplish this satisfactorily, and necessitates the skill of trained experts.

"In school practice tests have proved to be useful to administrators and supervisors, though the use which they have made of them has not always been discreet. Wherever teachers have been antagonized and pupils bored by tests, it is probable that tests have been used improperly. A highly valuable possibility of service for tests, it seems to me, has been but little recognized. Too often tests are given, the data are tabulated, conclusions drawn therefrom are utilized by supervisors, and methods are revised by teachers because of them; but the pupils who wrote the tests are not informed of any of the results except in those

[6] Thorndike, E. L., and others, "Standard Tests and Their Use," *Teachers College Record*, XXVI, pp. 93–94.

rooms where unsatisfactory conditions have brought about attempts to shift the blame to these pupils.

"Interest in school work and in personal efficiency therein can gain tremendously in the minds of many pupils by knowledge of the standings which they have achieved in various kinds of scholastic tests. From the fifth or sixth grade on, boys and girls can use to advantage charts such as those Dr. Trabue recommends in his article. The will to improve, growing out of the boy's study of his own record, and the will to self-mastery and self-management, are desirable outcomes that are at present not as commonly expected from testing programs as they should be. The final justification for every testing regime rests in Mary Jones and John Smith, and it therefore behooves all persons who are making and giving tests to take them into partnership as soon and as completely as is feasible."

What Teachers Think of the Tests.—Miss Helen Bailey of the Philadelphia High School for Girls states the attitude of many teachers toward standardized tests when she says:[7] "In sharp contrast to the modern content that is being urged, we are confronted by tests and measurements which demand the most time-worn kinds of mechanical skill. To develop the degree of facility which a child needs to enable him to adapt himself to a standardized test, where, to his confusion, a lengthy problem must be crowded into one inch of space, and his concentration is shaken by his awareness of the stop-watch, it is necessary to drill upon manipulations to a very unfashionable extent. So we lie between two great movements which trend not at all in the same direction."

Present Status of Standardized Tests

Where standardized tests are valid and reliable instruments, they may be profitably used for purposes of "general survey diagnosis," and even in some cases for class and individual diagnosis; but this work must be based more and more upon the cooperation of all concerned down to the pupils themselves.

In all fairness to these tests it should be said that they have gone beyond what Professor Woody calls the "curiosity" stage and "the stage in which the predominant idea was the use of tests

[7] Bailey, Helen C., "In Defense of a Few Old-Fashioned Things," *Educational Outlook*, II, p. 224.

for determining existing levels of achievement," and, in some respects at least, have approached the third stage, "in which the predominant idea is the utilization of tests as a means for the improvement of instruction."

Other New-Type Tests

Nobody will claim, of course, that the new-type tests which are not standardized will be a panacea for all of our testing ills. They also have their shortcomings as we shall see. As Professor Horn points out, we need to emphasize the method of testing instead of any one test itself. He lists [8] the important purposes of educational tests to which I have made certain additions and amplifications as follows:

1. To give in a brief period of time a rapid survey of all the skills or abilities to be tested. Pupils' difficulties with given topics are often due to neglect of the teacher to provide instruction in certain small details. What is mathematically to the teacher a single skill may be psychologically to the pupil a complex of skills.

The new-type test on the quadratic formula on page 379 will bring out this point in a practical way:

The old essay-type of examination tested only a few skills and it took a long time to do even that. The standardized test in mathematics has made little progress in this direction although it has made a start.

2. To remove the personal equation in marking papers; in other words, to make the scoring of tests objective. The marking of examination papers in the past has been highly subjective. This procedure is human. Every teacher is prejudiced with respect to his pupils. Objective tests can be marked in only one way and the record is impersonal.

As in the preceding case, the test on page 380 referring to optional historical information, that might be given in certain classes, can be scored in only one way.

[8] Horn, Ernest, General Preface of *Improvement of the Written Examination*, by G. M. Ruch, Scott, Foresman and Co.

THE QUADRATIC FORMULA $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Taking $ax^2 + bx + c = 0$ as the type form of the general quadratic equation which has the two roots x_1 and x_2, insert the proper values in the following table, using the numbered columns which correspond with the eight given equations, as shown for the first five in Ex. 1:

1. $x^2+3x+2=0$. 3. $x^2+9x-36=0$. 5. $x^2-8x=-15$. 7. $x^2-10x=11$.
2. $x^2-3x+2=0$. 4. $x^2-13x-40=0$. 6. $x^2+10x=-24$. 8. $x^2+14x=32$.

	1.	2.	3.	4.	5.	6.	7.	8.
$a =$	1							
$b =$	3							
$c =$	2							
$-b =$	-3							
$b^2 =$	9							
$4ac =$								
$-4ac =$								
$2a =$								
$b^2 - 4ac =$								
$+\sqrt{b^2 - 4ac} =$								
$-\sqrt{b^2 - 4ac} =$								
$-b + \sqrt{b^2 - 4ac} =$								
$-b - \sqrt{b^2 - 4ac} =$								
$x_1 = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} =$								
$x_2 = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a} =$								

Optional Historical Information

In the left-hand column is the name of some person, place, time, phrase, or people corresponding to some item in the right-hand column. Insert on the dotted lines the letter of that item which corresponds to the numbered entry in the left-hand column:

1. Thales.	.... *a.*	Square on the hypotenuse.
2. Pythagoras.	.... *b.*	First printed edition of Euclid's geometry.
3. Euclid.	.... *c.*	Great Egyptian surveyor. Formula $A = \sqrt{s(s-a)\ (s-b)\ (s-c)}$.
4. Greeks.	.... *d.*	Laid the foundation for geometric proofs.
5. Romans.	.... *e.*	Foundations of demonstrative geometry.
6. Plato.	.... *f.*	Euclid lived about this time at Alexandria.
7. Heron.	.... *g.*	Wrote a manuscript about 1600 B. C. on Egyptian mathematics.
8. Archimedes.	.... *h.*	Wrote the greatest of the Greek textbooks on geometry.
9. Alexandria.	.... *i.*	To them is due the best work in geometry in early times.
10. Ahmes.	.... *j.*	Plato lived in Athens about this time.
11. Arabs.	.... *k.*	A Greek city at the mouth of the Nile. Euclid lived there.
12. 300 B. C.	.... *l.*	Applied mathematics to practical uses, as in great engineering works.
13. 500 B. C.	.... *m.*	Translated Greek works on geometry, whence they reached Europe again through the Latin.
14. 400 B. C.	.... *n.*	Pythagoras lived in Croton, Italy, about this time.
15. 1482.	.... *o.*	Lived in Syracuse, Sicily. Computed a fairly good value of π.
16. Pons asinorum.	.... *p.*	First English translation of Euclid's great geometry.
17. Golden section.	.... *q.*	Badge of the Pythagorean brotherhood.
18. Pentagonal star	.... *r.*	The best known of the early translations of Euclid into Latin, about 1260.
19. Campanus.	.... *s.*	Angles opposite the equal sides of an isosceles triangle.
20. 1570.	.... *t.*	Dividing a line into extreme and mean ratio.

3. To show the pupil how efficient he has been. The old essay-type of examination did not do this. It was too narrow in scope to show much of the pupil's mastery of the field.

4. To show the teacher how efficient he has been. The essay-type of examination can reveal how well the teacher has taught only a few things. The modern educational tests through their diagnostic features set forth clearly the strengths and weaknesses of pupils so that the teacher can do remedial work when necessary. This is the strongest feature of such tests. When properly prepared, they reveal the pupil's knowledge and abilities with respect to every essential feature of the work.

5. To measure the value of a given text or method of teaching. The tendency today is to build up a course of study and then provide tests to determine how well the objectives are being realized.

6. To find out how long it takes to teach a topic. Today we do not know how long it takes to teach anything. We know that we are getting mastery on only a few things. With proper testing, for example, we could find out how long it takes to teach a normal group of ninth grade pupils to factor the difference of two squares to any desired degree of mastery.

7. To serve as teaching devices. No textbook can contain all the exercise material required by modern courses of study, particularly in the fields of oral and of rapid written work. The tests supplement the text and relieve the teacher of the necessity of supplying the additional material.

8. To find out what is a desirable content. No matter how desirable some of our objectives may be, it may be that some of them are not within the reach of the pupils we are teaching. It is only by intelligent testing that we can properly decide finally which objectives are valid.

9. To enable a pupil to rate himself on his performance in relation to his former record or that of his fellows. The first of the two schemes is probably the better to use. Just as a man is eager to beat his previous score in golf, so a pupil is usually eager to improve his score in mathematics.

10. To survey the status of teaching in a school system. These tests are designed to afford a comprehensive survey of the work of a semester or a year. They are often given by some one who is surveying the school system and, although general, are useful in indicating the status of teaching in the system.

Criteria for Constructing New-Type Tests.—The following criteria for constructing new-type tests are suggested:

1. Every test should attempt to measure a pupil's ability to master the subject matter that has been presented to him. This means that the one who makes the test must discern clearly the objectives in the topic or course and must build the test so as to measure the extent to which these objectives have been realized.

2. Every test should emphasize primarily those parts of the subject matter which are fundamental and to which the pupils have directed the most attention. In other words, the test must be comprehensive. Nothing should receive attention that is not worth perpetuating in the course. If these two points are secured, we may say that the tests are valid.

3. The scoring of each test should be so arranged that in scoring them all teachers may obtain exactly the same results, or the same teacher may on giving a test a second time obtain the same result that he got at first. The test should be so constructed that it is self administering. In other words, the tests must be objective.

4. Every test should be reliable, that is, it should measure what it measures to a satisfactory degree of accuracy.

5. Every test should be so constructed that it is possible to set some sort of standard of achievement for a pupil.

Illustrations of New-Type Tests in Mathematics.—Although there are many kinds of new-type tests in mathematics, space will permit us to illustrate only a few. They are chosen from intuitive geometry, arithmetic, algebra, and introductory demonstrative geometry and illustrate the most frequently used forms of these tests.

In a subject like mathematics we can, to be sure, make many unique types of tests which will help us to measure the achievement of pupils. Obviously we cannot take the space here to illustrate each of the varieties of such tests which a teacher can make when once he becomes interested in diagnostic and remedial work. Anyone who has made new type tests knows that it is a very fascinating piece of work which helps the teacher to become better acquainted with his subject, to learn more about his pupils, and thus to become a better teacher.

USING ARITHMETIC IN MAKING INVESTMENTS

Complete each of the following statements in such a way as to make the best arithmetic sense:

1. Savings banks frequently add the interest to the depositor's account every 6 mo., and the total amount (disregarding the cents) then draws interest. In 3 yr. a deposit of $1500 in a savings bank which pays 4%, adding the interest to the account every 6 mo., will amount to $.....

2. Mr. Roe invests his savings in bonds which have a face value of $14,000 and bear interest at 5¾%, payable semiannually. Every 6 mo. Mr. Roe will receive $..... from this investment.

3. In Ex. 2, Mr. Roe is able to buy the bonds at less than their face value, the bonds and the broker's commission ammounting to $13,685. The annual income from the bonds will represent a return, to the nearest 0.1%, of ..... upon the amount he invests.

4. If a man bought 25 shares of railroad stock, with a par value of $100 each, at 108¾ and paid his broker a commission of 15c per share, the total amount that he invested was $.....

5. After the man in Ex. 4 had owned the stock for 6 mo., the directors declared a dividend of 3¾% on the stock. The amount that the man received at this time was $....., which represents a return of approximately .....% a year upon the amount invested, as found in Ex. 4.

6. A man paid $6500 for a house and rented it at $45 per month. If his expenses the first year were $128.50 for taxes, $12.25 for insurance, and $86.50 for repairs, his net income from the house for that year was $..... If, instead of buying the house, the man had invested the $6500 at 6%, he would actually have been ..... off financially.

7. After owning the house for 5 yr., the man in Ex. 6 sold it for $7800. Taking the first year's rent and expenses as a fair average for the 5 yr., the average annual net income from the house, including the profit made on the sale, was $....., which represents a return of approximately .....% on the original investment.

8. If I had my choice of investing in (1) a bond paying 5¾% which can be bought at 97, including brokerage, (2) a mortgage paying 6%, and (3) a sound stock on which the average annual dividend rate has been 7% for some years and which can be bought at 109¼, plus a brokerage charge of 15c per share, I should select the ..... because it pays the highest rate of return.

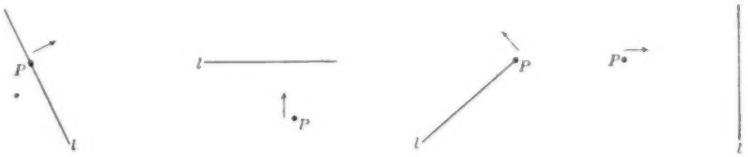
ANGLES IN EVERYDAY LIFE

If a statement in the following list is true, underline the word "True" in the column at the right; if the statement is false, or partly so, underline the word "False":

1. At the corners of a rectangular room the edges meet at right angles.	True	False
2. At 9:30 A.M. the hands of a clock form an obtuse angle.	True	False
3. In making one complete revolution, the spoke of an automobile wheel generates an angle of precisely 180° with its original position.	True	False
4. In moving from 12 o'clock noon to 3 P.M., the hour hand of a watch generates a right angle.	True	False
5. A tree growing vertically on the side of a hill makes right angles with the plane of the ground.	True	False
6. If one of three parallel cross-arms on a telegraph pole is perpendicular to the pole, the other arms form right angles with the pole.	True	False
7. In a garage with an A-roof the slanting planes of the roof make obtuse angles with the side walls of the garage.	True	False
8. A path which cuts diagonally across a square field makes angles of 45° with the sides of the field.	True	False
9. If two straight roads cross obliquely, the four angles formed at the point of intersection are equal.	True	False
10. A blade of an electric fan that is making 300 revolutions per minute generates an angle of 1800° in 1 sec.	True	False
11. A telephone pole standing vertically at the edge of a horizontal sidewalk makes right angles with the edge of the walk.	True	False
12. If one of two ships sails directly northeast from a harbor, and the other one sails directly southeast, the courses of the ships form an obtuse angle.	True	False

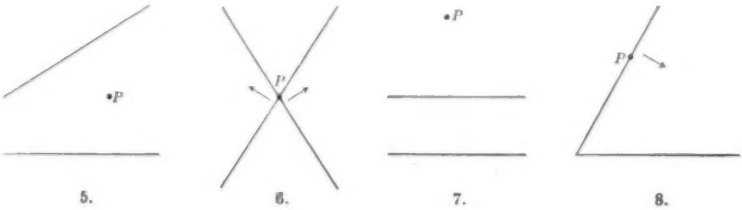
DRAWING AND CONSTRUCTING PERPENDICULARS

Using a ruler and either a protractor or a draftsman's triangle, in each of the following figures draw a line through P that shall be perpendicular to l in the direction shown by the arrow:



1. 2. 3. 4.

Using a ruler and compasses, and leaving all construction lines on the page, in each of the following figures draw lines through P that shall be perpendicular to each of the two given lines:



5. 6. 7. 8.

Using a ruler and compasses, and leaving all construction lines on the page, construct the perpendicular bisector of each of the following line segments:

9. 10. 11. 12.

ALGEBRAIC NUMBER SCALE

After studying the algebraic number scale shown below, complete each of the following statements:

1. If I start at + 2 and count 6 spaces in the positive direction, I reach ..... Since counting 6 spaces to the right is the same as adding + 6 to +2, the sum of + 2 and + 6 is .....

2. In adding + 5 to + 4, I start at ..... and count ..... spaces to the ..... I thus reach .....; that is, the sum of + 4 and + 5 is .....

3. If I start at — 2 and count 6 spaces in the negative direction, I reach ..... Since counting 6 spaces to the left is the same as adding ....., the sum of — 2 and — 6 is .....

4. If I start at — 2 and add + 6, I reach .....; that is, the sum of — 2 and + 6 is .....

5. If I start at + 2 and add — 6, I reach .....; that is, the sum of + 2 and — 6 is .....

6. Subtraction may be regarded as finding the number which must be ..... to the number subtracted in order to equal the other.

7. In subtracting + 4 from + 9, I may start at + 4. I then count ..... spaces to the ..... in order to reach + 9. Since direction to the right is ....., this shows that the result of subtracting + 4 from + 9 is .....

8. In subtracting — 4 from — 9, I may start at — 4 and count ..... spaces to the ..... in order to reach — 9; that is, the result of subtracting — 4 from — 9 is .....

9. In subtracting — 4 from + 9, I may start at — 4 and count ..... spaces to the ..... in order to reach + 9; that is, the result of subtracting — 4 from + 9 is .....

10. In subtracting + 9 from — 4, I may start at + 9 and count ..... spaces to the ..... in order to reach — 4; that is, the result of subtracting + 9 from — 4 is .....

11. The result of subtracting + 4 from — 9 is ....., and the result of subtracting — 9 from + 4 is .....

TRANSLATING ALGEBRAIC SHORTHAND

The right-hand column contains translations of the equations at the left. Match the two columns by writing in each square the letter corresponding to the proper translation:

1. $3x = 24$.	☐	*a.* When 2 is subtracted from a certain number, the result is 8.
2. $\frac{1}{2}x = 8$.	☐	*b.* Three times a certain number, decreased by 6, is equal to 15.
3. $x + 6 = 24$.	☐	*c.* At 25 mi. an hour, an automobile traveled 120 mi. in a certain time.
4. $x - 2 = 8$.	☐	*d.* A boy had 75c, but after buying a jackknife he had only 15c left.
5. $2x + 5 = 15$.	☐	*e.* When a certain number is multiplied by 3, the result is 24.
6. $3x - 6 = 15$.	☐	*f.* A boy had $1.20 in nickels.
7. $75 - x = 15$.	☐	*g.* The result of subtracting twice a certain number from 106 is 84.
8. $24 - \frac{1}{2}x = 15$.	☐	*h.* The result of adding 6 to a certain number is 24.
9. $\frac{1}{3}x - 5 = \frac{1}{4}x$.	☐	*i.* The sum of two consecutive numbers is 15.
10. $106 - 2x = 84$.	☐	*j.* When 5 is added to twice a certain number, the result is 15.
11. $12f = 84$.	☐	*k.* Half of a certain number is equal to 8.
12. $0.05n = 1.20$.	☐	*l.* The result of subtracting half of a certain number from 24 is 15.
13. $25h = 120$.	☐	*m.* When each side of an equilateral triangle is increased by 1 in., the perimeter of the triangle becomes 84 in.
14. $l + \frac{1}{2}l = 10$.	☐	*n.* The number of inches in the length of a rectangular flower bed that is f feet long is 84.
15. $3(s + 1) = 84$.	☐	*o.* When 5 is subtracted from one third of a certain number, the result is one fourth of the number.
16. $n + (n + 1) = 15$.	☐	*p.* A rectangle is half as wide as it is long, and the sum of the length and the width of the rectangle is 10 ft.

INFERENCES REGARDING PERPENDICULARS

In the following cases underline the expressions in small type which seem to you best for completing the statements, and then insert at the right the letters corresponding to the expressions selected:

1. If two lines in a plane are each perpendicular to a third line, the two lines are
   a. Equal. *b*. Complementary. *c*. Perpendicular. *d*. Horizontal. *e*. Parallel.

2. If two adjacent sides of a parallelogram are perpendicular, the other two sides are
   a. Equal. *b*. Supplementary. *c*. Perpendicular. *d*. Parallel. *e*. Oblique.

3. The shortest line that can be drawn from a point to a line is
   a. The bisector of the line. *b*. Oblique to the line. *c*. Parallel to the line. *d*. Perpendicular to the line. *e*. A perpendicular bisector of the line.

4. A line that is perpendicular to one of two parallel lines is
   a. Parallel to the other. *b*. Equal to the other. *c*. Oblique to the other. *d*. Equidistant from the other. *e*. Perpendicular to the other.

5. A line that is perpendicular to one of two perpendicular lines
   a. Is perpendicular to the other. *b*. Intersects the other. *c*. Is parallel to the other. *d*. Is equal to the other. *e*. Bisects the other.

6. The line which bisects the angle between two perpendicular lines is
   a. Oblique to both lines. *b*. Perpendicular to both lines. *c*. Parallel to one of the lines. *d*. Equal to both lines. *e*. Parallel to both lines.

7. A line that is perpendicular to a slanting line
   a. Bisects the slanting line. *b*. Is parallel to the slanting line. *c*. Is also a slanting line. *d*. Is horizontal. *e*. Is impossible.

8. A line that is parallel to one of two perpendicular lines
   a. Is parallel to the other. *b*. Is perpendicular to the other. *c*. Is oblique to the other. *d*. Bisects the other. *e*. Is equal to the other.

9. In a right triangle two of the altitudes
   a. Bisect each other. *b*. Are perpendicular to the hypotenuse. *c*. Are parallel. *d*. Meet outside the triangle. *e*. Coincide with the sides.

Present Weakness of New-type Tests.—We should not conclude without pointing out some of the weaknesses of new-type tests.

1. Many of these tests contain obsolete material of no possible importance.

2. Some of the tests are poorly arranged and badly printed.

3. Some tests, especially those made by busy teachers, have had all sorts of impracticable features, such as loose detached sheets, transparent paper, and the like, that make them unfitted for classroom use.

4. Certain speed tests tend to glorify the machinery of mathematics. Habits on this mechanical part of algebra may be overdone. Skill in algebra must not be obtained at the expense of understanding. There is no value in finding out how fast errors can be made.

5. Only of late has anything been done to test the field of algebra, and only recently has anything of importance been done in geometry, although a good start has now been made.

6. The tests do not measure attitudes, appreciation, and the like. This need not remain true, and at the present time steps are being taken to remedy this defect.

Conclusion

Finally, it is not contended that the new-type tests should replace the more traditional types, but it is suggested that a broader and wiser use be made of these newer instruments of measurement in supplementing the older ones. There is little doubt that the pupils themselves like the new types much better than the older ones. We know from actual use that larger areas of subject matter may be tested in less time by the new-type tests and that the drudgery of scoring is greatly reduced by their use. We have reason to believe that we obtain more information about the extent and quality of a pupil's learning through the use of the newer tests and that our remedial instruction is more intelligent and worthwhile.

In spite of the frequent inadequacy and inaccuracy of teachers' judgments both in setting good examinations and in marking them fairly, it should be more generally recognized that these same teachers are in the long run the ones best situated to do the task. Teachers can not only learn how to make objective tests that will have both measuring and diagnostic value, but they can also learn to use them intelligently. This ability to use the tests will increase in proportion to the progress that teachers make in understanding more scientific methods of measurement.

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STUDY OR RECITATION FIRST IN SUPERVISED STUDY IN MATHEMATICS CLASSES?

By HARL R. DOUGLASS

School of Education, University of Oregon

For twenty years the supervised study movement has been making slow but certain progress. Employed but in the occasional school ten or twelve years ago, it has found a place in a very strong minority of schools, which minority threatens to become a majority in another decade or so. The most common of its many forms is the division of the class hour into two portions, one of which is given over to supervised study and the other to recitation.[1]

In the large number of instances by far where the daily divided period of supervised study has been employed, study has followed recitation in the class hour. This sequence we may call the R–S sequence.

The Problem in General

From time to time, regardless of which method was being used, the question has been raised as to whether it would not be more effective to have the recitation period follow rather than to precede the supervised study period. Various theoretical considerations have been advanced for each procedure, and teachers may be found who will give testimony as to which is the superior. No precise reliable data have been available to show whether any real difference exists as to the efficacy of the two plans, and the problem has remained in the realm of speculation.

The Theoretical Considerations

On the other hand there are teachers who claim in favor of the "study-recite" plan that:

[1] The author is not committed to the divided period plan of supervised study. On the contrary, he is very much inclined to prefer, for the more capable teachers, the use of large units in the teaching, *e.g.*, problems, projects, challenges, contracts, etc., the completion of which would generally involve giving over several entire class periods in succession to study and possibly entire class periods to recitation.

1. Pupils will study more earnestly, with keener attention and less susceptibility to "mind wandering" and other distractions, if the study immediately precedes the recitation, while in the "study-recite" plan there is a natural tendency for the student to feel that since the recitation is over he will not be called to account for the material to be studied for twenty-four hours, and consequently he does not feel the responsibility for preparation during the class hour nearly so keenly.

2. The use of the study-recite plan will operate to correct the tendency of the teacher to permit the recitation to encroach on time which should be spent on study.

3. The imminence of the recitation will operate to reduce the disciplinary problem in the supervised study period, which in the "recite-study" plan has been in many classes not inconsiderable.

4. The first part of the lengthened hour is more favorable to effective study than the latter part, which follows half an hour already spent on the particular subject, and when the student has begun to grow tired of it. This point is urged particularly by elementary school and junior high school teachers.

On the other hand, the friends of the "recite-study" plan feel that:

1. If the study period precedes, the natural tendency is for the student to postpone beginning his preparation until the supervised study period, and that for this reason insufficient time will be spent in preparation.

2. Even if it is granted that pupils will have spent some time in study before the opening of the period, the fact remains that the pupil should begin his preparation under supervised study conditions and complete it independently and the supervision is most valuable in getting the pupil safely started on his work.

3. The learning of material with a view to reciting immediately does not favor more permanent attention; it operates to lessen the value of the recitation as a means of determining what the pupil has really learned.

4. The pupil should begin his study with the assignment, the material covered in the previous recitation, and the problems remaining from it fresh in mind.

The author of this paper determined to put the question to the experimental test. He was able to employ ten pairs of sections of pupils in the University of Oregon high school for the experiment. Three of these pairs were classes in mathematics, one pair in each of the three years of junior high school, and it is with these three pairs that the following paragraphs will deal.

Controlling Other Factors

Every effort was made to hold constant or to equalize every significant factor likely to affect the progress of the two sections

except the experimental factor, *i.e.* the difference in sequence employed.

All pupils likely to be concerned in any of the experiments were given the Otis Group Intelligence Scale (the advanced examination). The first year or seventh grade section was given the Stanford Achievement tests in arithmetic, and the second and third year sections were given the Douglass algebra test. The Otis test scores were taken as measures of general mental ability and the Stanford and Douglass scores as measures of aptitude in mathematics. Composite scores were made up from the two sets of test scores by the method of equal variabilities.

The pupils to be assigned to each pair of sections were then paired off so as to make each pair consist of pupils of approximately equal composite intelligence and mathematics test scores and chronological age. One from each pair was assigned to one section and the other from each pair to the other section. In doing this, care was taken to see that the resulting sections were of equal average composite test scores, equal initial test score, and equal average age, and, in addition, equal in variability in the first two of these respects as measured by the standard deviation.

Because of excessive absence of one of each pair, a few pairs were dropped from the experiment.

In all cases both sections of each class met at the same hour in the day.

Rooms and teachers were assigned to the experimental sections so as to hold constant those factors for each pair of sections. In order to equalize what differences may have existed between the rooms or teachers of any two sections, these were exchanged at the conclusion of the first half of the term.

Co-operative and trustworthy teachers were obtained for the experimental classes. The work in "practice teaching" is under the supervision of the director of the experiment and the head of the department concerned and much importance is attached to the recommendation given to the appointment bureau of the school of education by these two people. The instructors were caused to feel that their success was not dependent upon the outcome of the tests given to either of the two classes which each taught, but upon doing excellent teaching in the classroom in harmony with the experimental conditions as judged by the head of the department and the director.

Each teacher was required to record daily the absences of pupils and any interruptions of any sort which tended to interfere with the work of the hour. In every instance in which one section was affected by any such interruption as the shortening or omission of a period due to assembly, fire drills, etc., the other section meeting at the same time was similarly affected. The principal of the high school notified the teachers in advance whenever it was necessary to shorten a period, and the teachers were instructed to distribute the time so deducted equally between recitation and study.

The final data upon which the conclusions from the experiment are drawn do not include the scores made by any pupil absent as much as ten days more than the individual paired with him.

No effort was made to control the teaching methods employed by the teachers of the experimental classes other than that in one section where the supervised study was employed, it should take place during the latter part of the hour and that in the corresponding section it should precede the recitation or discussion and that the amount of time given over to supervised study should be approximately equalized between the two sections. Socialized recitations, question and answer methods, lectures, visual instruction, reports, and other types of class procedures were used as the instructor desired. However, as in every case both instructors in paired sections were working under close supervision of the department heads and as the subject matter was practically identical, the methods used in any two paired sections were quite similar.

No one supervised study technique was used uniformly, but the practice in this respect was a quite natural one. Each teacher employed whatever procedure seemed most effective—at times silent, individual study; at times group, socialized study; at times class discussion study. As in the case of the recitation there tended to be agreement in methods employed from day to day between paired sections for the reason that the two instructors of paired sections planned their work under the direction of the same supervisor. In the main the supervised study was of the silent individual type.

Measurement of Relative Progress

Tests were administered for the first few and the last few days of the term as a means of determining the relative progress of the two sections of each pair. The tests for the seventh and eighth grade sections were constructed by the head of the mathematics department under the supervision of the author and with the assistance of the instructors of the experimental sections. They were constructed to be as nearly objective as possible and to test only that part of the year's work covered during the experimental period. For the third year sections, the Douglass Algebra tests were used. The final tests were similar in form and general content.

The reliability coefficients of the initial and the final tests are given below in Table I.

TABLE I

Reliability of the Initial and Final Tests *

	N	Initial Tests		Final Tests	
		r_{11} *	P.E.	r_{11}	P.E.
Mathematics J2	35	.791	.048	.878	.028
Mathematics J4	33	.670	.079	.708	.068
Mathematics J6	29	.801	.050	.900	.025

* Calculated by the Spearman Brown propheey formula from the correlation between scores made on the odd and on the even exercises of the test.

In all three pairs of sections the section in which the S-R sequence was employed made greater gains as shown in Table II.

TABLE II

Mean Gain and Difference in Mean Gain of the Pupils in the Three Pairs of Sections

	Mean Gain S–R Sec.	Mean Gain R–S Sec.	Difference in Favor of R–S Sequence
Seventh grade	16.11	14.61	1.50
Eighth grade	21.44	21.20	0.24
Ninth grade	11.91	10.77	1.14

Those familiar with statistical procedure realize that the obtained gains from studies of this sort may be the results of a

real difference in the experimental factors, chance errors of measurement and of sampling, or systematic errors from those sources such as might occur in the selection of experimental groups or in the failure to rule out or hold constant some factor favoring one group or the other. As has been pointed out, great care was exercised to prevent the operation of such systematic errors. We may readily ascertain to what extent the differences obtained are beyond the realm of chance. We may safely assume that the distribution of chance errors is "normal." We already know the differences themselves.

The standard deviation of these mean differences may be calculated by the conventional formula for that purpose.[2]

The chances that the differences are real rather than the results of a limited sample or other chance error are given below:

TABLE III

CHANCES IN 100 THAT SUPERIORITY IS IN THE DIRECTION OF THE OBTAINED SUPERIORITY

	d	SD_d	$\frac{d}{SD_d}$		Chances in 100
Mathematics J2	1.50	.82	1.83	RS > SR	97
Mathematics J4	.24	1.56	.16	RS > SR	56
Mathematics J6	1.14	.96	1.19	RS > SR	88

At the close of the experiment all instructors engaging in it were asked to write out and incorporate with the "log" an opinion as to the relative efficiency of the two experimental factors. It is interesting to note to what extent these opinions are borne out by experimental results. A comparison is furnished in Table IV.

TABLE IV

AGREEMENT OF EXPERIMENTAL RESULTS IN JUDGMENT OF INSTRUCTORS

Class	Experimental Results Favoring:			Teacher Judgments Favoring:		
	R–S	Neither	S–R	R–S	Neither	S–R
Mathematics J2	x	x		2	2	
Mathematics J4						
Mathematics J6	x			1		1

[2] See Kelley, T. L., *Statistical Method*, p. 182, formula 140, or consult index of any of the better treatises of statistical method.

The Relative Effectiveness of the Two Sequences for Pupils of Different Intelligence Levels

It is quite often asserted somewhat confidently and dogmatically that the S–R sequence is better adapted to the more intelligent students who are not compelled to do as much outside study as are the less gifted and who will be more likely, because of their superior intelligence, to give their studies the necessary attention to be able to complete their preparation satisfactorily before the beginning of the class period. It seemed desirable to throw some light upon this question of relation of sequence to intelligence.

That neither method is better adapted than the other to any given level of ability is shown by the facts that the correlation coefficient between the superior gain of the pupils in the S–R sections over their mates in the R–S section and the intelligence scores for all three pairs of sections was only .115 with a probable error of .088 and that the correlation short reveals no significantly superior achievement for either group of students at any particular level of intelligence.

Conclusions and Interpretations

The experimental data show clear-cut tendencies, the explanations of which are not easy to furnish. It is significant that the results in mathematics classes consistently favor the R–S sequence, though to be sure, the superiority of that sequence in the case of mathematics J4 was slight and not significant.

It is very likely that assignments in form subjects, and particularly in mathematics are usually of such a nature that the student profits most by beginning work on them as soon after the work is explained as possible and by beginning under supervision and with help at hand.

The Validity of Teachers' Judgments of the Relative Effectiveness of the Two Sequences

While not conclusive because of the small number of teachers involved, the data employed in the study furnished adequate basis for a very firm tentative conclusion that teachers' judgments of the relative effectiveness of teaching methods of approximately the same merit are not to be relied upon with great

confidence, even under the conditions of very closely controlled experiment.

Relation of Intelligence to Method

It seems quite clear that neither plan is peculiarly adapted to the more gifted or the less capable students.

Limitations of the Inferences Which May Be Made From Data of Such a Study

From the data and legitimate conclusions of a study of this type, it is very easy to make inferences which do not take into account variations in the conditions which may wield very appreciable influence upon the relative effectiveness of the two experimental sequences. The safety with which such inferences may be drawn depends upon the quality of the logic and experience of the person making the inference. It is within the range of possibility that even in other schools using different textbooks or different courses of study that the relative effectiveness of the two sequences may be reversed. This, however, seems a remote possibility. It should be evident that when an inference is made from a conclusion based upon restricted conditions allowance should be made for the fact that a very large number of conditions will be certain to be somewhat different, as between the experimental classes and the classes concerned in the inference, *e.g.*, location, time, physical plant and equipment, teaching and student personnel, etc. In order to make any application at all of generalizations in any field of study, the judgment of the individual making the inference must be called into action to determine to what extent the variables not constant (time, location, equipment, etc.) are of such a nature that differences in them are likely to influence the operation and the relative effectiveness of the two sequences. With these fundamental considerations in mind, the applicability of the conclusions of this study appears quite wide-spread, assuming that reservations appropriate to changed conditions are made.

ABILITY GROUPING IN MATHEMATICAL CLASSES

By FLETCHER DURELL

Belleplain, N. J.

One of the most striking outcomes of the recent wide use of intelligence tests and the various new-type tests in educational work has been the revelation of the unexpectedly wide differences in the ability and aptitudes of pupils. Consequent upon this revelation has come the demand that the instruction given to pupils shall be more exactly adapted to their individual capacities and needs.

Surrounding and underlying this whole movement has been the increasingly thorough study of efficiency in all lines of activity, higher as well as lower, and the insistence that education as well as other social processes shall be made as finished and productive as possible.

Where special individual instruction of each pupil is not possible, the new movement asks that, as far as possible, pupils, when studying a given subject, shall be segregated into three sections representing different levels of ability, viz: sub-average, average, and above-average groups. This three-fold sectionizing may be regarded as a first step toward the utmost possible degree of individual instruction.

It is important at the outset to form a broad conception of the basis of this matter of the three-fold sectionizing of a class or grade. It is customary to speak of these sections as determined by three levels of natural ability in pupils. But the conditions involved in actual class-room practice are much wider and deeper than this one matter of innate mental ability. It would be unfortunate and erroneous to form the idea that the pupils in the lowest or primary section of a class are all to be there because of inferior mental endowment. For some pupils might be in such a section because of time lost by sickness, or because of defects in early training, or as a result of some early prejudice formed against a study. Some bright pupils may want to learn in the class room only the minimum essentials of a certain topic or branch, and prefer to use in some other

way the time saved by limiting their work to what may be termed the first stage of mastery.

To cover these broader conditions, descriptive terms like the following might be used for the three sections of a class: retarded, average, advanced. Letters are also in a measure noncommittal, and A, B, C (A being for the advanced section), or X, Y, Z (X highest) are often used.

The writer prefers wherever possible to describe the work aimed at in the three sections as "three stages of mastery," the most elementary stage being regarded as covering what is often called the minimum essentials of a subject. The phraseology "three levels of ability," however, is used so often as to have a flavor of standardization, and it seems necessary to use it sometimes even when the wording "three stages of mastery" would be more accurate and helpful.

The actual application in the class room of the principle of ability grouping or three stages of mastery involves two leading problems:

I. The selection and organization of the subject matter to be studied. This includes both the writing of adapted text-books and the framing of curricula, as well as, of course, a number of minor details.

II. The class-room administration of the material thus organized.

Organization of Material

In the process of organizing material, the first and in some respect the most fundamental problem that arises is that of forming groups of examples concerning any given topic adapted to the instruction of the three sections of pupils in a given class. This often takes the form of a three-fold grading of each exercise in a book. In meeting this need, we are aided by certain standard categories or methods which together form a kind of science or technic.

The first of these principles is that of *number*. Thus, in teaching the addition of integers, the number of addends in each of the examples assigned for the first stage of mastery (that is, assigned to the pupils of the C group) should be smaller than in the examples given to the B or A pupils. For instance, if there are four addends in a C example, there might be six addends in a B example, and eight in an A example. This prin-

ciple applies also in teaching other kinds of addition as of fractions, or decimals.

In getting a C, B, A, scale of difficulty in examples and problems the category is to be applied also to the number of items in a problem, the number of steps and processes in its solution, the number of abstract or technical words used, of special business or engineering technics referred to, or of results to be determined as answers.

So in a completion example we get a rising scale of difficulty by having one, two, or three blanks to be filled in by the pupil. Similarly in a matching example, or in the selection of correct answers we may deal with a small or large group of items.

Also in an exercise given to the low ability group, the number of different types of examples should be small, with much repetition of each of these types.

In applying this principle to the solution, for instance, of equations in algebra, we must consider the number of terms in an equation, the number of unknowns in a system of equations, the number of steps in the solution, and the number of results called for.

So in geometry in proving a book theorem or an original, we pay attention to the number of items in the hypothesis, of steps in the process of reasoning, and of relations to be discovered or proved.

Closely related to the category of number is that of size or *magnitude*. Thus in distributing examples according to their difficulty, we must consider the size of the various kinds of numbers involved, as the size of the denominators of fractions, the number of figures (or places) in integers or decimals, the degree of equations, the degree of difference between successive examples or types of example, and so on.

Another category or principle to be considered in this connection is that of the concrete as contrasted with the *abstract*. It is evident that in large degree the work on a given topic assigned to the pupils doing elementary work should be as concrete, everyday, familiar, and non-technical as possible. At the opposite end of the scale will be work so technical, general, or unfamiliar that it may call for the use of the dictionary, encyclopedia, or for other research work. A valuable intermediate stage is that where the concrete and abstract are combined as in the following:

Ex. 1. If sugar costs 6 cents a pound, tell how you would find the cost of a given number of pounds.

Ex. 2. If the length of a given rectangle is l feet and the width 5 feet, express the area.

The degree of difficulty of a problem may also be made to vary according to the *language* in which the problem is expressed. Thus in stating a problem for the first stage of mastery (that is, for C pupils) the sentences used should be short and direct, the words simple and familiar, and each statement full and complete. For the abler sections, inverted and implicit forms containing hypothetical clauses may be used. Some of the words and terms may be unfamiliar and calling for the use of the dictionary.

The category of *order* is often important in this connection. Thus in arranging the first stage or C examples, some simple and evident plan should be followed. Several examples of the same type should succeed each other with slight changes in form and contents. After examples of one type have been given, other types can be treated in like manner and finally a few mixed examples of different types can be given. So ordinarily in the wording of an elementary example, the order of statements made should correspond as closely as possible to the order of operations to be performed in the solution; this implies, of course, that the data, or what is given, are to be stated first, and afterward what it is required to find.

In examples calling for initiative and *self-activity* on the part of the pupil, in some problems the degree of originality required may be so slight that such problems are adapted for use with *C* pupils; in fact, such examples will be little more than completion examples of a simple kind, thus:

Ex. 1. A woman bought 2 pounds of coffee at — cents a pound. Fill in the blank with a probable number and work the example.

Ex. 2. Make up and work an example concerning the purchase of 5 pounds of rice at a grocery store.

Corresponding examples for *A* pupils would be:

Ex. 1. Make up and work a problem concerning the purchase of three different articles at a grocery store and the change received from some bill.

Ex. 2. Collect as many different kinds of receipted bills as you can and state the differences between them.

Ex. 3. Collect the facts concerning the invention and adoption of the metric system and write an essay of not more than 500 words concerning them.

Of other particular kinds of difficulty in grading examples, special mention should be made of that obtained by requiring the pupil to find some function of the customary form of answer. Thus, the following are problems of this kind adapted to A pupils:

Ex. 1. If the hypotenuse of a right triangle is 10 and a side is 8, find the area of the triangle.

Ex. 2. If the perimeter of a rectangle is 40 feet, and the length is 8 feet, make a scale drawing of the rectangle.

Ex. 3. If $3x - 5 = 7$, find the value of $7x + 4$.

As some part of what is required in such examples is omitted, they may be described as *syncopated* examples.

If branches of mathematics are taught on the co-operative or closely inter-related plan, examples based on these functional relations can be graded according to the degree and complexity characterizing such relations.

Over and above the above-listed categories of difficulty used in the gradation of work, there are other more subtle elements of graded difference hard to analyze, but whose presence must be recognized and allowed for.

Evidently also several of the above sources of dffiiculty may be complexed together in a single problem in various ways.

If the examples to be worked by pupils are selected and grouped in the three-fold way just described, then the organization of the rest of the subject-matter in a given branch will be greatly facilitated. This remaining matter includes descriptions and definitions of terms and concepts, explanations and illustrations of rules and principles, and in general what is often called the theory of the subject.

If in teaching a given topic the examples in the exercise to be worked by the pupil are graded in the triple way described above, the explanatory matter preceding this exercise may be made brief and elementary. For much of the development of the simpler rules and principles can be put in the shape of easy examples which are a part of the C part of the exercise. Also the more complex and technical concepts and principles can be inserted in the course of the rest of the exercise largely as problem work for the more advanced pupils. Thus only the prime essentials of the theory of the subject or topic will be told to the pupil. The rest he will work out in large measure for himself.

A leading educational authority has recently stated that at present no educational principle is more generally accepted and advocated than the importance of learning by doing, while at the same time no pedagogic principle is more generally violated. Nowhere in the educational world are the ideal and the achievement further apart. If ability grouping and instruction can be made successful, it will go far to remedy this defect in our methods and to achieve the corresponding ideal. In this connection it is to be remembered that in this age of the automobile, radio, moving picture, and the like, pupils are year by year growing more and more impatient of mere explanation and extended talk, and anxious as soon as possible to get busy doing work for themselves.

The next matter that naturally comes up is that of the organization of subect-matter into curricula or courses of study adapted to three-fold-ability instruction of classes. We have room only to consider what is probably the most general and fundamental problem in this connection. This problem can probably be presented and discussed to best advantage in connection with the teaching of some special process as the teaching of long division in arithmetic. When long division is first taught to a class graded in the three-fold manner, shall the topic be omitted altogether for the lowest ability or C pupils (and some other work substituted), or shall an attempt be made to have some of the work on the given topic so elementary that all three ability sections of pupils can begin the study of the topic at the same time? The writer is of the decided opinion that if the above categories of difficulty for the grading of examples be used, some work so easy and primitive can be devised that all pupils can begin the study of a topic at the same time. Thus for the first instruction of C pupils in long division, groups of examples like the following could be used:

$$21\overline{)441} \qquad 21\overline{)462} \qquad 21\overline{)483}$$

Even with higher pupils it will often be found advantageous to have them work rapidly at the outset many such easy examples.

Where topics are developed progressively in successive semesters the general plan of forming a curriculum would be to slide back the semester assignments in the way illustrated in the fol-

lowing tabulation for the two semesters in each of grades four and five.

	C	B	A
IV 1.....	p	q	r
IV 2.....	q	r	s
V 1.....	r	s	t
V 2.....	s	t	u

In this diagram the work to be done on some topic, as long division, in the fourth and fifth grades is denoted by the letters p, q, r, etc. Thus the work assigned to the B section in the first semester of the fourth grade (or q) will be given to C pupils in the next semester (IV 2), and so on.

Class Room Administration

If textbooks were written and made available and courses of study were organized in the ways suggested in the preceding pages, the question next arises as to the best methods of class administration, that is of actually using such material in the class room.

There are two principal ways in which pupils may be grouped for such instruction:

I. Pupils may be separated into C, B, and A sections (called homogeneous grouping) where C pupils are taught as a separate class and expected to attain only the first or primary stage of mastery; the B pupils taught as another separate class and expected to attain the second and higher stage of mastery; and of course, the A pupils as a section which is expected to acquire the third and still higher degree of mastery.

II. The use of alphabetical or mixed sections.

Taking the first of these, if a textbook be used in which the material is organized as described with the examples in each exercise distributed in three graded parts and with much of the explanatory and theoretical matter inserted in the exercises in the self-active form, the preliminary or explanatory work of the teacher with a C class on a given topic would include not only a discussion of the given definitions and illustrations but also the

working on the blackboard (in part by the pupils) of certain of the examples in the lesson to be assigned. This lesson assigned for seat or home work would cover the examples in the first (or C) section of the exercise, and the statement would be added that extra credit would be given to a pupil for doing any of the B or A examples in the exercise.

The same kind of explanation might be given to a B section of pupils, though more briefly, and the class be asked to work as a lesson, the odd (or even) examples in section C of the exercise and all the examples in Section B, with extra credit for any examples solved in the A part of the exercise.

With an A section of pupils all of the examples for the first stage of mastery in an exercise can usually be treated as oral or sight work. The written or seat work will consist of the examples in the B and A parts of the given exercise, a few of the more difficult problems being treated as extra credit work.

If, instead of three sections, it is desired to distribute pupils in two or four or more ability sections, the methods just described can readily be adapted to meet the situation. Also with experience and study, some approximation to individual instruction along with certain group methods is attainable.

It is understood, of course, that from time to time, there is to be a redistribution of pupils so as to keep the sections as homogeneous as possible.

We consider next the teaching of a mixed or alphabetical class or section.

Sometimes the number of pupils in a given grade is too small to make it possible to divide and teach them as separate A, B, C, ability groups, and it is necessary to instruct them as a single heterogeneous section. Even when a choice between the two methods is possible, some teachers prefer the alphabetical or mixed section because of certain advantages to be discussed later.

How then are textbooks and curricula organized on the C, B, A basis to be used with a mixed or heterogeneous section? The answer is two-fold. On the one hand the alert and capable teacher can with some modifications use the methods suggested above for segregated sections. But if the teacher prefers not or is unable to do this, he can regard the given textbook simply as composed of exercises graded with unusual care. A teacher can vary lessons in several ways from year to year corresponding

to the special characteristics of different classes. All pupils in a class will start at the same place and work the same examples, but the B and A pupils will naturally go further and extra credit work may be assigned as a possibility for all alike, though some differentiation in this respect is possible for the skillful teacher.

In this kind of class instruction, the separation of the pupils into ability groups exists only on paper or in the teacher's mind. The pupils themselves may not be aware of it. But most of its advantages may be realized.

Other points in the technic of this administration will be given when discussing the advantages of this method of class instruction.

Besides the two principal methods of organizing classes for graded A, B, and C instruction which have just been described other possible methods are as follows:

(1) The segregation of deficient pupils into a special section and the division of the rest of the pupils of a given grade into alphabetical sections.

(2) The segregation of the abler pupils into a special class and the division of the rest alphabetically.

But after what has been said no special discussion is here necessary as to the technic of class room instruction in such cases.

In this connection, the question has sometimes been raised as to how matters are to be managed where teachers are required to grade all pupils with respect to some single numerical standard as 100 and to hand in written reports for all on this basis, the passing mark for instance being 60. The writer has used ability grouping in his classes for thirty years and has much of the time been required to hand in reports in the way just described. It has been his custom to give a C pupil who has been doing satisfactory work a grade of 60, and if he does superior work (including some of the extra credit work assigned), a grade as high as 70 is possible. A pupil who has done satisfactory or exceptionally good work in a B section can receive a grade of from 80 to 90; while a pupil in an A section can receive a mark of 100 or 100A. This method is also followed in grading general examination tests which must fit in with the general administration of the school as a whole and often have extra-mural relations as in preparing pupils for College Board examinations.

However in tests given for private purposes from day to day

the writer follows a somewhat different method. The first part of such a test usually consists of examples of C and B difficulty, while the last two or three examples are of A difficulty and are described to the class as "extra credit work." Any pupil who solves all of the first part of the paper (the C and B examples) correctly gets on that paper a grade of 100. This work of 100 here is supposed to represent not absolute perfection, but an attainment like a bogey score in golf. If he also solves some of the extra credit examples, he may get a grade of 110 or 120. In making out his final term report (where the maximum credit is limited by school regulations to 100) any grades above 100 which a pupil may have obtained during the term may be used to help make up deficiencies in other tests or in obtaining an added distinguishing letter as A, this being a kind of *cum laude* or *summa cum laude* honor.

Especial attention should here be called to the term "extra credit." For special work of this kind, the writer at first tried descriptive terms like "optional examples," or "optionals," or "honor work," but found that these gave only a moderate stimulus. The words "extra credit" make almost a revolutionary difference. The idea that a pupil has done all that may fairly be expected of him and that he is going on to heights above this 100 percent mark has an especial appeal to him. A grade above 100, as 120 or 130, also has the advantage of suggesting to him an endless vista of possible further achievements, while the old idea of a mark of 100 as representing finality and perfection often had a stunting and limiting effect on the pupil's ideas of possible or desirable further achievement.

Advantages of Different Methods

It will be useful at this point to make a somewhat detailed statement of the respective advantages in the two main class room methods of using ability grouping in teaching a subject, viz: segregated or homogeneous grouping, and the use of mixed classes; and afterward a more general statement of the advantages in ability grouping in all cases.

If the more able pupils in a class be selected and taught as a separate A section, such pupils will not be held back while the weaklings are receiving extra help from the teacher. Also time can be saved for them by treating certain parts of the text as

oral work or by passing over it rapidly in other ways. Hence by this method, bright pupils should be able to cover far more ground, gain a far higher and more thorough scholarship, and receive an intellectual development fitted to make them leaders in the outside world in later life.

To offset this, however, we have the fact that unless a teacher is especially alert, naturally bright pupils may feel that after being placed in a special A section and thus stamped with the mark of approval they do not need to do very much more work, with the result that they settle down into a kind of complacent inertness.

However all agree that in a democracy such as ours there is a tendency to reduce all minds and personalities to one common level and one uniform pattern, and that special efforts should be made to develop distinguished personalities. Along with a democracy of opportunity we should have an aristocracy of merit and special ability, kept in a fluid state. If wisely used, ability grouping in school classes can be made a means to this end, or at least a step in this direction.

If an A pupil or section does not need to study the enriched course in a branch in the way just described they can cover the main essentials rapidly and thus complete their study, say, in one half the regular time, and then use the time thus saved to cover the essentials of some other branch.

Correspondingly if the pupils weak in a subject are taught as a separate C section, it is claimed that they will not be discouraged at the sight of brilliant work done easily by others, that they can receive simplified, adapted instruction, experience the joy which comes from real achievement, and thus make much greater progress than they otherwise would.

Also among the pupils in each of the segregated groups there should be a deeper and more definite competition, and the advantages which come from a wholesome rivalry. On the other hand, experience has shown that there are special advantages connected with giving A, B, C instruction in a mixed or alphabetical class, and that a teacher is therefore not to be discouraged or deterred from using such instruction even in this somewhat limited form.

In a mixed class, the brighter pupils by watching the strange and unanticipated errors and crude processes of the less gifted

pupils, and the repeated corrections of these errors, gain a much more thorough and many-sided grasp of the prime essentials of a subject than they would if working alone. This all round study plus a study of a few difficult technical, specimen developments or applications of a principle is often of greater value to the bright pupil than a more complete study of the technical development of this principle without the more thorough foundation.

The mastery gained in a mixed class gives a greater development of the quanity-sense values of the subject, and especially of the important efficiency power whereby when the prime essentials of a subject have been thoroughly mastered, other parts and details will largely take care of themselves.

It has been objected that the use of graded A, B, C instruction will tend to produce a caste system which is foreign to our American social ideals. With the right kind of administration any such tendency, if it really exists, should be easily avoided. In any case in a mixed class, there will be far less of it present. Indeed, if, as should be the case, bright and successful pupils are asked occasionally to help retarded ones and the benefit of the superior pupils constantly pervades the whole, democracy and aristocracy will permeate and aid each other in a highly efficient way.

As to the C pupils, the fear that they may be depressed and discouraged if required to recite with more gifted pupils in a mixed class is apt to be exaggerated. If matters are rightly handled, such companionship may be made on the whole helpful to the C pupils. The sight of superior work being done may be a stimulus and inspiration rather than a discouragement. The methods, ambitions, and spirit of success of superior students will percolate and diffuse themselves. The use of the term "extra credit" has an especial value here. When a pupil of moderate ability realizes that 100 does not mean absolute and complete perfection, but rather a bogey attainment possible to any average pupil, and that extra credit is possible beyond it and is daily achieved often by many, not a few B and C pupils will throw in every ounce of power that they have to get some of this treasure of achievement open to all.

The fact that in an alphabetical section the classification of pupils in A, B, and C groups is registered only in the teachers

mind and is in a state of constant flux, is an aid in this respect.

We may conclude therefore that segregation of pupils into A, B, C sections reciting separately is not essential and is a secondary consideration; that ability instruction in its most important features can be carried on without it. The writer may even add that he has had considerable experience in both methods and that, on the whole, with his present light, he prefers mixed to segregated classes. In his recent book, "The New Leaven," Stanwood Cobb makes a statement to the same effect.

In conclusion it may be well to make a brief statement of the advantages which characterize ability instruction, no matter what plan of class room administration be followed.

In the first place, we get in some measure work adapted to the ability and tastes of the individual pupil. Students are not required to attempt the impossible and thus are less likely to form a distaste for the subject or for mathematical or abstract reasoning in general; on the contrary, with reasonable skill on the part of the teacher each pupil acquires at least some elementary knowledge of the subject, and all should go much further in this mastery than they would if the instruction were not thus adapted to them individually.

Especial mention should be made of the satisfaction, if not joy, which pupils thus get in their work and the added values which come from such pleasure. This joy in their work often seems to arouse in them certain natural growth processes which carry them forward without much effort on the part of the teacher.

Highly adapted instruction thus arouses self-activity in a vital and comprehensive form. The heuristic attitude and urge thus developed often help to lift the C pupil to the B or A level.

By the deep-seated self-activity thus aroused, both teachers and pupil are aided in certain pedagogic details which are often distasteful to the pupil and therefore difficult for the teacher.

Thus, reviews of the work of preceding grades are in large part put in the form which enlist the co-operative self-activity of the pupil and are made more effective in various ways.

Theory, often distasteful to many, is divided up and given in small units to be assimilated largely by the self-active method, it being left to the brighter pupils later to collect and organize, perhaps with some help, the separate items into a systematic

whole. Organized A, B, C instruction is an aid to the teacher in other ways. Diagnosis of pupils, discovery of their individual difficulties and weakness, and needed remedial work are made easier. In fact the use of exercises and other material graded in the A, B, C style as recommended furnishes a combination of more or less constant diagnosis and cure.

Also a deeper and more vital pedagogic research is made possible for the teacher. For in C classes the mental educative processes of the primitive mind are spread out in a detailed form, often twice as long as usual in the past and correspondingly full of detail, for investigation and study.

Moreover, the teacher is aided in more special ways as in the collection and invention of adapted examples. Again, if the teacher has to prepare pupils for a college where only honor pupils are accepted, the teacher finds the materials and methods ready at hand for such preparation.

Hence we may fairly say that instruction by ability grouping, even as at present understood and developed, combines the best principles of individual instruction with the best of group instruction and the two are combined in a way to aid and improve each other.

THE PLACE OF MATHEMATICS

"It is not only possible but necessary that mathematics be applied to psychology; the reason for this necessity lies briefly in this: that by no other means can be reached that which is the ultimate aim of all speculation, namely *conviction.*"—*Herbart.*

"For success in the life imposed on him he needed, as afterwards appeared, the facile use of only four tools: Mathematics, French, German, and Spanish. With these he could master in very short time any special branch of enquiry and feel at home in any society. These four tools were necessary to his success in life, but he never controlled any one of them."—*Henry Adams.*

"The more advanced the sciences have become, the more they have tended to enter the domain of mathematics, which is a sort of center towards which they converge. We can judge of the perfection to which a science has come by the facility, more or less great, with which it may be approached by calculation."—*Quetelet.*

POPULARIZING PLANE AND SOLID GEOMETRY

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Popularizing geometry is indeed a broad subject and the methods employed depend on the individual teacher, her class, the locality in which she teaches, and the particular situation in which she finds herself. In fact, methods vary from year to year, constantly reaching out toward the goal of popular geometry. But in order to have something concrete on which to work, I am suggesting methods which have at various times helped me to relieve the monotony which frequently overwhelms the most enthusiastic of classes. The development of these suggestions I leave to the very fertile imaginations which you as geometry teachers must possess.

In the ordinary class of plane geometry there are three types of pupils. First, there are those who take geometry because they like mathematics. How enjoyable would teaching be if all our classes were composed of boys and girls of this type; yet how little stimulus to do the unusual! Second, we find in our classes those who must have the course for college entrance credit. And the third type consists of students who have drifted in because their friends are there, or simply because they need another subject to fill up their schedule. The first type we are not concerned with here, for it would take a very poor teacher indeed to kill the interest which these people have for geometry. But when I speak of popularizing geometry, I am thinking primarily of the students who are spending a year studying a subject for which they, upon entering, have no burning interest. We, as geometry teachers, should have something to offer them. We have no right to expect wide-awake boys and girls to sit in our classes five days a week for forty weeks without deriving some lasting benefit. And to derive benefit, they must be interested.

To secure and maintain interest in geometry, we must popularize it, first through the teacher, the only live element in classroom instruction. She must be vitally interested in her subject,

else how can she make her pupils like it? If she does not constantly show these boys and girls, who will probably never take any more mathematics, that she has something worthwhile to offer them, she should give way to the teacher who can. She will never be missed.

If a teacher is wide awake, she will gain interest by making apt references to the history of geometry. And they must be apt, for the history of mathematics can be made exceedingly dry for these young folks. For example, I often place on my bulletin board at the beginning of the year a placard which reads, "There is no royal road to geometry." The pupils eye it curiously without saying anything. Then one of the first assignments that I give them is to look up the life of Euclid from books in the library, impressing them early with the immensity and the antiquity of the "History of Geometry." After this assignment, the eyes that gazed wonderingly at the placard, open wider, and some bolder student says "I know who wrote that," pointing to the card on the bulletin board. It is not hard then to launch a discussion concerning Euclid, and some pupil usually bursts forth with the idea that there is no "royal road" anywhere. They particularly like the story of the boy who had to be given a few pennies because he must gain something concrete for studying geometry. And the conclusions that they draw will often surprise one.

A picture of Euclid (although I have been told there is really no picture of our Euclid) on the bulletin board, a little later in the year, will invariably call forth the remark, "Yeah! That's the guy who is responsible for all our misery." And the funny part of it is, that the boy or girl who makes that remark rarely means it. You will find, without much effort, that your students write, "Poison," "Take in Moderate Doses," "Brain Whirlpool" and the like on their Latin and history books as well as on their geometry books. Don't you remember, as a student, how you liked to make your teachers think that everything you had to do was distasteful? But that remark about geometry causing us misery ought to call forth from the wide-awake teacher a veritable stream of talk on the purpose and value of studying geometry. Some students will take joy in the satisfaction you can show them which results from problem-solving. Go deeply into the fact that geometry is the foundation of many professions,

and get your students to picture a world with no knowledge of it. Perhaps you can get a remark from the student who started the discussion to the effect that "If Euclid hadn't written his textbook, some one else would."

Aside from the accuracy and the power of reasoning that every good teacher hopes to develop, teach the student to strengthen his powers of observation and appreciation of geometric form throughout the course. Teach him that geometry brings him in contact with absolute truth as no other subject can. Teach him that geometry is a universal science. There is no boy or girl but is interested in the proposed attempt to signal to Mars by by constructing a huge figure to represent the Pythagorean Theorem on the Sahara Desert. How would geometry studied on Mars differ from that studied here?

In addition to letting the pupils see that she is interested in geometry herself, to making pert references to the history of geometry, and constantly throughout the course emphasizing the purpose and value obtained in studying geometry, I think that a teacher can make her subject popular by often indulging in mathematical recreations. Perhaps it isn't strictly geometrical to ask your classes what would happen if you dropped a ball into a hole which had been dug through the earth. Most of the class, after a little thinking can describe its movements fairly correctly, and will finally follow it to rest at the center. But interesting is the discussion if you have a student who thinks further than the rest. One such person came forth with this contribution. "What would happen if the object you dropped were weighted?" Well, what would happen? After a time, the class arrived at the conclusion that the object would turn over every time it passed the center. But this did not entirely satisfy. The ball mentioned came to rest at the center. Would this object? Such a problem not only stimulates interest, but it also develops reasoning powers, unless the student possesses the mind of one of mine, who said, "Well, could you actually dig a hole clear through the earth?" He was answered in the negative, and told that we were merely discussing the principle of what would happen if such a hole existed. "Well," replied he of no imagination, "I can't see the use of discussing something that can't happen."

And let me just remark here; don't hesitate to bring up prob-

lems of this sort for fear that you can't answer all questions asked. Remember, the student respects you more if sometimes you admit you don't know everything. When you guess at an answer, tell him you are merely speculating. He will be more ready to abide your decisions when you make them. He knows you are sincere.

If you are interested in books on Mathematical Recreations, may I suggest "Mathematical Wrinkles" by Sam Jones, "Mathematical Recreations" by Ball, and "Scrapbook of Elementary Mathematics" by White. The last contains an interesting account of how Alice wandered into Wonderland after studying mathematics.

The next item which aids us in popularizing our subject is a textbook which is good to look at, and easy to handle. Perhaps it doesn't make any difference to you if your book is brown and ugly and contains no pictures, or is bright colored and shining and full of pictures showing you the geometry of everyday life. But it makes a difference to your pupils. And if you have a textbook that the pupils like, you are going to have pupils who will do more studying. An ideal text should contain constant challenges to the pupil's originality, and should make him hunt for things. Don't make him wait until the end of the year before you allow him to read the last pages in his book. Make want to find out how this particular story is coming out.

The textbook which stimulates interest contains frequent summaries, and creates each theorem as a tool which can be used. Frequently take ten minutes off, and tell your pupils where they are going. Give them the essentials of the course, show them where to find the summaries in their books. Help them sum up what they have already studied, and skim through the next five or ten pages with them. They like to look ahead. Tell them of things which you think may be easy or hard for them. They love to disagree with you after those parts are over, but by being frank, you give them confidence in you, and create in them a desire to study and perhaps show you whether certain parts are hard or not.

The third, and probably most important group of popularizing methods will be found in your classroom procedure. Naturally a wide-awake teacher and attractive text make for an interesting class. It should be an outcome of the first two. We

often find in our classes pupils who always seem to know the answers to our questions when we call on them, but who never willingly volunteer to contribute anything to the class. One can often reach these students by selecting some reliable pupil in the class to keep daily record of the people who take part in the recitation. My secretary has one kind of a mark for those who volunteer, and another for those who recite merely because they are called on. This not only stimulates the pupil to greater effort, but informs the teacher which students are being slighted. At the end of each week this record is posted on the bulletin board.

We all agree that one of the greatest benefits to be derived from the study of geometry is the power of logical reasoning that it develops. We still further agree that many students pass through a year of our instruction, having developed very little of it because of their dependence on other children. When they attack originals, they copy their class work, and fail on their tests. I have found that by teaching pupils to work together at the board and at their seats, I have overcome, to a large extent, both the copying and the distaste for originals. In fact, my pupils do better work on problems than on the theorems worked out for them in their books. Their ingenuity is challenged and they like it.

I often put a good and a poor student together at the board working for an entire class period. Of course the good student learns more by teaching, but the poor student receives a stimulus to be more like his partner as well as added knowledge and methods of attack from him. After the teacher has explained until she can explain no more, the student teacher can often find the difficulty with alacrity, and clear away the mistake. Sometimes I put two groups of very good students near each other, and they fairly race to beat each other. Again I put two students together, the type of students we used to call lazy, although we have been told recently that there are no such students. I invariably do this after these two students have been working with others who have carried them along with their work. Then these students whom we once called lazy, when working together often become disgusted with each other, and exhibit a little more ambition. You notice I say "often." Some of these we never reach, but I am thankful for a method which reaches even a few.

Variations of this program are many and useful. I try usually to put students together who like each other and like to work together. Fine results ensue. Students by this method learn "how" to work together, and hence their work becomes the result of their own thinking.

In your class room you can constantly stimulate imagination, interest, and the desire to read and investigate through apt references to subjects they know little about. The unknown is always attractive. For instance, solid geometry can be referred to in the study of locus problems, to exercise the imagination. A discussion of the fourth dimension always revives waning interest in a class. Sam Jones in his "Mathematical Wrinkles" has written an interesting article on it. One also appeared in the MATHEMATICS TEACHER for March, 1926. There are many things in astronomy which are of interest to a student of geometry, such as the shapes of the heavenly bodies, their orbits, and methods for finding their distances away. Your discussion may wander away from the geometrical, but you will return refreshed. A crude conception of zero and infinity provides fuel for thought. Your pupils will like to ponder on the minuteness of zero, and to conceive of infinity as a point so far way that they can't think of one farther. Let them understand that mathematicians think of this as the point where parallel lines meet. It is good for your "A" students to realize that they have not achieved the zenith of knowledge simply because they are "A" students. Frequently show them how elementary is their knowledge.

I believe in establishing throughout the course a connection between plane and solid geometry. I also believe in very early forming a connection between algebra and geometry. We can expand geometrically $(a + b)^2$ and can work complementary and supplementary angle problems by algebra. No matter how much students groan about their algebra inwardly they are thrilled, in a new subject, to find something they know.

Long lists of problems should be given on the contract basis, with frequent individual corrections. This allows the student to travel at his own rate of speed, and helps the teacher to acquaint herself with the mental processes of her pupils.

Make a game of construction problems. Most students like to use their compasses and figure out constructions, provided they

are not troubled too much with their proofs. Since there is more or less monotony in formal demonstration which cannot be avoided, constructions rightly taught, at rather frequent intervals, provide very welcome breaks, and aid in popularizing geometry.

At the end of the course in geometry, a definite effort should be made to give the student a notion of what solid geometry is about. The construction of the five platonic solids at this time helps to connect the two branches of geometry and acts as a final advertisement for those who may be inclined to elect solid geometry.

In the fourth place we can make geometry extremely interesting by teaching throughout the year an appreciation of geometric form. Your text-book should aid you here. But whether it does or not, you can fill your bulletin boards with appropriate pictures illustrating geometry in machines, buildings, bridges, snow flakes, flowers, and gardens. Encourage your boys and girls to make original designs, to hunt for a honeycomb and a hornets' nest that they may find the hexagons constructed there and speculate on why the bee and the hornet make hexagons rather than squares or circles. Correlate your geometry with the biology department and you will have brought in to you flowers, and star fish, and even articles your wildest fancies had not imagined contained geometric figures.

Have a style show. In any class you will find enough geometric sweaters and sox, and ties and dresses and scarfs for the most exacting of your observing pupils. And the interest you will create!

Even in geometry, field trips are possible. An Institute of Arts provides a wealth of geometric figures. A trip into a snow storm may result in a snowball fight, and hence solid instead of plane geometry, but only after the beauty of the snowflakes has been discovered. A machine shop or factory will provide enough circles and their properties to last throughout the course.

Then as a culmination of this appreciation of geometric form which has been stressed throughout the course, require a notebook or scrapbook to be handed in by each pupil as a result of his geometric observation. I personally feel that if I have taught one of these students to be more observing, and thereby more appreciative of the things which exist in the world about him, I

have contributed something to his happiness and well being. Even if he never studies any more mathematics after he leaves my class, he has received something that is worthwhile. And since I have often received the best notebooks from students who do not go to college, I am sure that in a measure my purpose has been accomplished.

I have attempted to show how, through the teacher, the textbook, classroom methods, and the teaching of appreciation of geometric form, we can make plane geometry more interesting and hence more popular. To create a Utopia where the routine of geometry can be done away with is not my purpose. Mix in with things which are apt to prove monotonous something which is live and vital. That is what I am trying to say. There are many nooks and crannies that need filling. And yet if I filled them in my way, they might not meet your situation. If the suggestions I have made prove of any use to you in your geometry classes, I shall be very grateful for the opportunity of having brought them to you.

Solid geometry does not provide the problem of plane geometry in that the majority of the class are electing it because they have a fondness for mathematics. Of course, some few drift in because they want a semester's credit, but they are almost too few to be considered. However, no matter how interested a class may be when started, lack of preparation on the part of the teacher may quite easily turn it into a boresome procedure.

Solid geometry is often called a course in "Space Imagination" since we have to stress that so much throughout the semester. Before we begin the study proper, we can profitably start our work in imagination by mentally constructing plane figures for theorems, and following through as much of the proof as we can. As we transfer our thoughts to solid figures, we should use quantities of crude models, at first, gradually eliminating them, however, until we use them in only extreme cases.

Hand in hand with imagining the space relations goes the drawing of these same relations, and interpreting them after we have drawn them. Most students like to draw, and take pride in making figures which "they can see." Encourage them as well, to draw crude figures, free hand, for the more of these they try on the fly leaves of their books, the better will be their more carefully drawn figures.

Before we begin our study of prisms, pyramids, cones, and cylinders, I find it advisable to spend about a week constructing paper solids. I find this aids their mental conception of these figures, and thereby their drawings. *Observational Geometry*, by Campbell, published by Harper and Bros. in 1899, though not a recent book, has given me a great deal of help in these constructions. The Platonic Solids we construct at the very beginning of the term as an appetizer, if we have not already constructed them at the close of our Plane Geometry. When we have made a pyramid and a prism with the same base and altitude, we resort to the sand pile, and prove very concretely, much to the amusement of all concerned, that the "Volume of a Pyramid is one third the base times the altitude."

Field trips are not unheard of in solid geometry. The Institute of Arts provides cylinders, and cones and spheres, and many other objects for geometric speculation. We have a swimming pool in our school which we spend about a half hour measuring, and another two hours breaking up into prisms that we may find its volume. Local gas tanks in the city, both cylindrical and spherical, provide much material for volume problems.

Since solid geometry is an advanced course in mathematics, we should constantly challenge the students to do the unusual. Inspire them to construct one or more of the four star solids, sometime after they have made the Platonic Solids. One of my boys this year, in the midst of constructing a small stellated dodecahedron, decided he wanted to become a cave-man. So, instead of placing the twelfth point on his dodecahedron, he contrived a long handle, and after painting the affair to resemble wood smeared with blood, had for his courting as vicious a looking club as any pre-historic man could desire.

A slide rule makes for interest in geometry, as does use of the trigonometric tables and logarithms. They aid in long computations in area and volume problems, and by their use, we often enable a student to use something he already knows, and make him see the inter-relation between courses.

Correlate solid geometry with algebra. Develop $(a + b)^3$ geometrically. Encourage imagination through the extension of the two co-ordinate axes to three axes, and let the students draw figures illustrating these axes. Show your class how to locate a point in space, and then carry the development on into

the fourth dimension in thought, if nowhere else. Teach Conic Sections when studying the cone. Show where the name "conic section" comes from and give some practical applications. Introduce the student to analytic geometry, if you please. If he cares enough for mathematics to elect solid geometry, he will be eager to be exposed to something higher.

Martha Clark, of the Jefferson Junior High School in Cleveland in an article published in the Mathematics Department of the *High School Teacher* inspired me to make the experiment with eleventh and twelfth graders, which she made on junior high people. It worked! We combed the library for articles on primitive dwellings, and found many which contained pictures. Among them were the Eskimo igloo, the Indian tepee, the cliff dwelling, and South Sea Island hut, all representing solid geometry figures. In two weeks' time, we presented very clever models of each at a geometry house-warming. The Eskimos were very up-to-date and had a radio aerial in their back yard, and the Indians, a "Keep off the grass" sign, but the students in the midst of their fun, had found solid geometry principles in their houses. One of the girls ransacked the ten cent store for modelling clay, and constructed a very pretty set of cliff dwellings. She is now studying architecture at the University of Michigan.

The formal proofs of theorems are not the most important parts of solid geometry. Neither are they the most interesting. They provide merely the framework on which the course is built. If we emphasize the importance of drawing accurate figures, their interpretation, and the fun involved in space imagination, at proper intervals; transfer our class to the construction of models when need be; constantly emphasize the beauty in the world about us viewed through solid geometry; expose the student to the marvels which the mathematical world has yet to offer him; constantly remind him to use the tools which his mathematics has thus far taught him; we will have a semester which is full of profit and pleasure for us as well as for our students. And what is more, we will be actually teaching solid geometry.

HOW MATHEMATICAL CLUBS AND ASSOCIATIONS MAY BECOME AFFILIATED WITH THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Under the leadership of Miss Marie Gugle, former president of the National Council of Teachers of Mathematics, certain important questions relating to the nature and work of the Council are answered for those who are interested.

1. *What is the National Council of Teachers of Mathematics?*

The National Committee on Mathematical Requirements showed the teachers of the country the need of a permanent organization that would unify the many local and sectional groups of teachers of mathematics and would carry on the work and spirit of that Committee. In 1920 the National Council of Teachers of Mathematics was organized in Cleveland, Ohio, by representatives of several associations. The Council is designed to be a *national federation of all organizations of teachers of elementary and secondary mathematics.* Such organizations may be clubs in small or large cities, large groups in neighboring towns or cities, departmental sections in connection with state organizations, or still larger associations representing several states or regions of the country. At the time of the publication of the Report of the National Committee, there were as many as one hundred such organizations, large and small. It is highly desirable that all of these and new ones should be federated in the National Council.

2. *What is the relationship in the general plan of organization between the National Council and local clubs or sectional associations?*

The National Council in no way replaces such local or sectional organizations. On the contrary, its purpose is to strengthen such clubs and associations by giving them opportunity to cooperate with similar ones in other sections of the country and to encourage the formation of additional associations in those sections of the country not yet organized. Through such a federation as the Council, some of the strong, well-established sectional associations, such as the Central As-

sociation or that of the New England States or of the Middle States and Maryland, can help promote the cause of better teaching in Mathematics and help raise the standards generally throughout the country.

3. *How may an existing club or association become a Branch of the National Council?*

First, the club or association should pass a resolution stating that the group desires to become an official Branch of the Council and authorizing the officers of the group to transmit the request to the Secretary of the Council. With this request, a list of members should be forwarded with the names of those already members of the National Council clearly indicated; or, in case of the larger organizations, a statement of the approximate number of members and the number belonging to the Council.

The Executive Committee of the National Council will act promptly upon all applications for the organization of Branches and will notify the petitioners accordingly.

Secondly, the officers of the group should encourage members so far as possible to join the National Council and to remit the $2.00 which pays for membership for one year and which includes subscription to the official journal, THE MATHEMATICS TEACHER. Thirdly, a list of the officers of the group should be filed with the Secretary of the Council. This list should be kept up to date at all times.

4. *How may a new group become affiliated with the Council?*

In a city, state, or section, any number of interested persons may organize and submit a petition to become a Branch. After the members of the group decide upon the territorial limits and elect their officers, the procedure is similar to that stated above. The national officers are ready at all times to cooperate in the organization of new local associations. In sections not densely populated, it may be advisable to have bi-state Branches. Such a Branch was organized in 1927 in Louisiana and Mississippi. It meets in connection with the Louisiana-Mississippi Section of the Mathematical Association of America. Such regional or state branches may be subdivided for more frequent sectional meetings than are possible for the larger groups.

5. *Is it necessary that an existing organization change its name in order to become a Branch of the Council?*

No, an existing organization need not change its name or lose

its identity in order to become a Branch of the Council. It should keep its name and merely append to the name a descriptive title; as, for instance, the following:

New York Mathematics Club
Branch of the National Council
of Teachers of Mathematics

Instead of the words "Branch of" a club may use the expression "Affiliated with."

6. *What is the relation between the Council and a Branch in matters of finance?*

Each member of the Branch who joins the Council must pay to the Council the annual dues of two dollars ($2.00), which includes a subscription to the official journal, THE MATHEMATICS TEACHER. Each Branch is free to finance its local expenditures in any way it may determine, usually by a small assessment on its members.

The By-Laws of the Council state that it is not obligated to pay the expenses of its Branches. However, upon petition of a new group desiring to become a Branch, the Trustees may authorize some officer or deputized representative to assist in the organization of the Branch at the expense of the Council.

7. *What is the relation between the Council and a Branch in matters of meetings and programs?*

Each Branch is free to organize its own affairs and to determine the number and character of its meetings. However, since the Council holds but one national meeting each year, at the place of and immediately preceding that of the Department of Superintendence of the N. E. A., it is greatly to be desired that all Branches hold as many local meetings as are feasible.

The secretary of each Branch should send to the *Editor* of THE MATHEMATICS TEACHER a schedule of its meeting dates and, after each meeting, should send a brief report for publication.

8. *How are the Branches represented in the Council meetings?*

All members who attend the National meeting are entitled to vote. It should be the aim of every Branch to send at least one official delegate, and as many more as possible, to each National meeting. Such a delegate would be responsible for bringing back to the Branch a full report of the Council meeting.

9. *What are the official publications of the Council?*

Each member of the Council receives the official journal, THE MATHEMATICS TEACHER, the only magazine published that deals solely with the teaching of elementary and secondary mathematics. No progressive teacher of mathematics can afford to be without the help of this journal. The Council also publishes Yearbooks, which are outstanding publications in the field of mathematics. Thus far three Yearbooks have been published, as follows:

First Yearbook (1926)—A General Survey of the Progress in Elementary and Secondary Mathematics of the Last Twenty-Five Years. Price $1.00, 210 pages, paper cover. (Out of print.)

Second Yearbook (1927)—Curriculum Problems in Teaching Mathematics. Price $1.50, 297 pages, in cloth, or $1.25 unbound. Bureau of Publications, Teachers College, 525 W. 120th Street, New York City.

Third Yearbook (1928)—The Teaching of Significant Topics in Mathematics. Price $1.75, 276 pages, in cloth. Bureau of Publications, Teachers College, 525 W. 120th Street, New York City.

The magazine and these Yearbooks furnish much helpful program material.

10. *Who are the officers and directors of the Council?*

Officers elected for 1928–29:

President.....HARRY C. BARBER
76 Court St., Exeter, New Hampshire

First Vice-President.....C. M. AUSTIN
Oak Park High School, Oak Park, Illinois

Second Vice-President.....MARY S. SABIN
Denver, Colorado

Secretary-Treasurer.....J. A. FOBERG
California, Penna.

Executive Committee

1929, ELIZABETH DICE, Dallas, Texas
1929, J. O. HASSLER, Norman, Oklahoma
1929, FRANK C. TOUTON, Los Angeles, Calif.
1930, WM. BETZ, Rochester, New York
1930, WALTER DOWNEY, Boston, Mass.
1930, VERA SANFORD, Lincoln School, New York City
1931, HARRY ENGLISH, Washington, D. C.

1931, MARIE GUGLE, Columbus, Ohio
1931, EDWIN SCHREIBER, Ann Arbor, Michigan

Committee on Official Journal

W. D. REEVE, New York City, *Editor and Business Manager*
JOHN R. CLARK, New York City
H. E. SLAUGHT, Chicago, Illinois

Under the new By-Laws, the incorporated council is under the control of a Board of sixteen Directors, consisting of the above officers, the executive committee, and the committee of Official Journal.

ANNUAL MEETING OF THE NATIONAL COUNCIL

The annual meeting of the National Council of Teachers of Mathematics will be held at the Hotel Statler in Cleveland, Ohio, on February 22 and 23, 1928. The general subject is "The Place of Mathematics in Education." An excellent program is assured by President Barber. Fred M. Burroughs of the John Adams High School in Cleveland, who is president of the Cleveland Mathematics Club, is chairman of the local committee on arrangement. It is hoped that before this meeting every mathematics club in the country will become a branch of the National Council and that so far as possible each of these branches will send a representative to the Cleveland meeting.

NEWS NOTES

THE Detroit Mathematics Club has already begun its meetings for the current year. The first meeting was held at Central High School on October 4. Dr. John R. Clark of The Lincoln School of Teachers College spoke on "Aims and Purposes in the Revision of the Mathematics Curriculum." In their first bulletin of the year which is sent to every mathematics teacher in Detroit and adjoining cities they give the National Council a good boost. We hope that all the other clubs will also help in the drive for 10,000 members by 1930. The general theme for the year in Detroit is "Curriculum" and it does not relate wholly to mathematics.

THE association of chairmen of mathematics departments in all of the New York City high schools at their first dinner meeting of the year on Saturday, October 6, voted to send Mr. John

Swenson of Wadleigh High School to Cleveland in February to represent them at the annual meeting. Mr. Swenson is known best to teachers of mathematics as a leader in the field of progressive mathematics, his most notable work perhaps being his development of a course in calculus for the high school. A resume of this course is published in the Third Yearbook of The National Council.

THE autumn meeting of the Mathematics section of the Association of Teachers in the Middle States and Maryland will be held at Atlantic City on Saturday morning, December 1, beginning at 9:30. A further announcement of the place and program for this meeting will appear in the December number of THE MATHEMATICS TEACHER, which will be mailed in time to reach all members of the Council before December first.

THAT greater interest and enthusiasm is being manifested in many groups of mathematics teachers throughout the country is manifested by the appearance recently of such publications as the Mathematics News Letter published in the interest of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics. The subscription price is $1.00 per year and the editors are S. T. Sanders of Baton Rouge, La., and Henry Schroeder of Port Hudson, La.

EVEN the students in some of the high schools throughout the country are editing papers dealing with the interests of the mathematics department. One of the most interesting ones that have come to the notice of the editor is The DeWitt Clinton *Mathematics Journal* prepared in mimeographed form and distributed among the students and faculty in the DeWitt Clinton High School of New York City. The story on page entitled "Suppose There Were No Mathematics" is quoted from the September issue of the above journal. Two prizes are offered to high school students for the best composition of 500 words or less upon the topic above. Such work appears to have possibilities that departments of mathematics may wish to consider seriously.

SUPPOSE THERE WERE NO MATHEMATICS

By R. F. CLARK

Henry Jones, section 519, sat at home in his room studying his Math. 5. His father, a successful C.E., president of one of the large construction companies, stepped into his room and asked how his work was going. Henry said that it wasn't going very well. He couldn't see why he should study mathematics. "I don't need it. It hasn't any value." His father replied, "Think it over. Suppose there were no mathematics. What about our civilization?"

* * *

Henry heard his mother calling, "Son, I think it is time to get up and start for school. My clock has stopped and I don't know just what time it is." Henry, dressing rapidly, turned a moment to call Central and get the correct time. After a disturbing delay, Central reported that they did not know. They 'guessed' it was 'about 8:10.' Henry wondered why they were guessing. He recalled how his father recently had explained the accurate way in which time is determined by mathematics, how the calendar is fixed. There flashed through his mind the remark of his father, the previous night, "Suppose there were no mathematics. What about our civilization?" He became convinced that in some mysterious way mathematics and the accurate determination of time had been done away with.

He found the subways tied up and long lines of cars, crowded but stationary, on the elevated. He hopped aboard a slowly moving trolley and tendered the conductor a dollar bill. This brought the greeting, "That's a dirty piece of paper. What've you got that's worth something?" Catching sight of a new pencil, the conductor said, "I'll let you ride for that pencil of yours." The old days of barter had returned. His father's words came back to him, "Suppose there were no mathematics. What about our civilization?" Slowly his trolley neared the school. As he entered the building, one late squadman started to take his name. A second said, "He isn't late. It's only 8:30." Henry left them arguing and hastened to his section room.

His section officer sent him to Room 210 to have his program changed. The Program Committee was not functioning. It could not. Program making needs mathematics.

He went to chemistry for his first period. It was chemistry very unlike that to which he had become accustomed. It was descriptive only. There were no formulas, no balancing of equations. These had gone with mathematics. As he came from this period, a friend from a physics class remarked, "Physics is no good. We couldn't find out to-day what the ratio of the diameters of a wheel and axle must be in order that an effort of 150 pounds may support a load of one ton. We tried a lot of ratios but none was the right one." Then Henry recalled a frequent remark of his father, "Mathematics is the handmaid of the Sciences."

During lunch period, Henry telephoned home to get late details about meeting his sister who was returning from Europe. His mother was worried. No word of her ship had been received and Henry was startled as he thought that navigation is impossible without mathematics.

On his way home he passed one of the skyscrapers that his father's company was building. No work was being done and Henry knew that all over the world work had stopped on new buildings, bridges, tunnels, etc., and before long there would be no such things, for the old ones would wear out and could not be replaced.

Someone was shaking him. "Wake up, Sonny. It's five o'clock. You've fallen asleep over your Mathematics. Mary has just telephoned from Hoboken that her boat is in. Father wants you to get her in the car." On the trip to Hoboken he kept thinking, "Suppose there were no Mathematics. What about our civilization?"

The following article appeared in *Mathematics News Letter* for September, 1928:

Mathematics in Every Thing

The enormous increase in industrial and scientific specialization which has taken place in the last quarter of a century has resulted in greatly increased demands on mathematics. This fact has strongly impressed the intelligent lay mind. (See in this issue of the News Letter a quotation from an editorial of the Saturday Evening Post.) The evidences of it are found

on every hand. For instance, the opening paragraph of the preface of a recent text for chemistry students * is as follows: "Science is best expressed when it is mathematically expressed; and yet most students of natural science cringe at the prospect of higher mathematics. It is because the author has seen so many students struggling against the handicap of an inadequate preparation in mathematics—spending more effort in avoiding mathematics than needs to be spent in learning it—that this book has been written." In similar manner one field after another is being compelled to turn to mathematics for that precise formulation of its basic principles which is essential to its ranking as a science or near-science.

In singular contrast with this rapidly growing demand for the technique of mathematics in scientific and quasi-scientific fields is the apparent attitude of indifference to it on the part, not only of many of our college-of-liberal-arts groups, but also of large numbers of those whose very choice of a major subject predestines them to the use of much mathematics in order to proper success in their major. Happily for the cause of mathematics, the latter group early discovers that any compromise which permits a basic tool to be unused makes that success questionable. On the other hand the case for mathematics as a valuable part of a proposed language or social science course has yet to be made out before the bar of opinion in a considerable number of our American institutions.

We seriously doubt if one percent of the trained mathematicians of America question the great value of mathematical study as a mental discipline. The fact that probably less than one-fourth of one percent of the membership of the Mathematical Association of America and the American Mathematical Society would be found questioning such a value, could at best be made to influence the construction of school curricula in considerably less degree than curriculum building has already been influenced by ideas, centering about "formal discipline," which have been fostered in the last quarter of a century by certain schools of education. The reason which is ordinarily advanced to account for this is that a mathematician is naturally if unconsciously biased in favor of his own field. On the other hand it should be evident to all fair-minded investigators that the existence of such a bias cannot justify a wholesale throwing overboard of the weight of EXPERIENCE.

—*S. T. S.*

* "Mathematical Preparation for Physical Chemistry," by Farrington Daniels, Associate Professor of Chemistry, University of Wisconsin.